

## Exercise Sheet 1

### Exercise 1

Let  $G_1$  and  $G_2$  be topological groups. Show that a homomorphism  $h : G_1 \rightarrow G_2$  is continuous if and only if it is continuous at the identity  $e \in G_1$ .

*Note the sharp contrast to arbitrary functions from  $G_1$  to  $G_2$ .*

### Exercise 2

(a) Let  $\Lambda$  be a closed subgroup of  $(\mathbb{R}, +)$ . Show that either

- (i)  $\Lambda = \{0\}$ ,
- (ii)  $\Lambda = \alpha \mathbb{Z}$  for some  $\alpha \in \mathbb{R}_{>0}$ , or
- (iii)  $\Lambda = \mathbb{R}$ .

(b) How many subgroups of  $(\mathbb{R}, +)$  are there?

*Note, that there is a wealth of both closed subsets (think Cantor set) and (arbitrary) subgroups of  $\mathbb{R}$  (part (b)). However, (a) shows that closed subgroups of  $\mathbb{R}$  are quite regular.*

### Exercise 3

Let  $X$  be a compact Hausdorff topological space. Show that  $\text{Homeo}(X)$  is a topological group when equipped with the compact-open topology.

*The result of this exercise may be seen as another argument for why the compact-open topology is a “good” choice.*