Exercise Sheet 2

Exercise 1

Consider the locally compact Hausdorff group $G = (\mathbb{R}^n, +)$ where $n \in \mathbb{N}_0$.

(i) Show that $\mathrm{Aut}(G)$, i.e. the group of bijective homomorphisms which are homeomorphisms as well, is given by $\mathrm{GL}(n, \mathbb{R})$.

(ii) Show that $\mathrm{mod}_G : \mathrm{Aut}(G) \to \mathbb{R}_{>0}$ is given by $\alpha \mapsto |\det \alpha|^{-1}$.

Note, that the automorphism group of G as an abstract group is much larger than the automorphism group of G as a topological group. Also, note that Δ_G is trivial since $(\mathbb{R}^n, +)$ is abelian whereas mod_G is not.

Exercise 2

Let G be a locally compact Hausdorff group. Show that the modular function $\Delta_G : G \to \mathbb{R}_{>0}$ is continuous.

It is possible to extend the continuity of Δ_G to continuity of mod_G using a certain topology on $\mathrm{Aut}(G)$.

Exercise 3

Let G be a locally compact Hausdorff group with left Haar measure μ. Show that for all $f \in C_{00}(G)$ we have $\int_G f(x^{-1})\Delta_G(x^{-1})\ d\mu(x) = \int_G f(x)\ d\mu(x)$.