

Exercise Sheet 6

Exercise 1

Review the proof of Cartan's Theorem.

Exercise 2

Review our examples of (topological) homogeneous spaces with a focus on the smooth structures involved.

Exercise 3

Let G be a Lie group with Lie algebra \mathfrak{g} . For every $g \in G$, let

$$\text{int}(g) : G \rightarrow G, x \mapsto gxg^{-1}$$

denote conjugation in G by g . Define the adjoint representation of G by

$$\text{Ad} : G \rightarrow \text{GL}(\mathfrak{g}), g \mapsto D_e \text{int}(g).$$

Furthermore, define the adjoint representation of \mathfrak{g} by

$$\text{ad} : \mathfrak{g} \rightarrow \mathfrak{gl}(\mathfrak{g}), X \mapsto [X, -].$$

Show that $D_e \text{Ad} = \text{ad}$.