$$(f_x)_y(0,0) = \lim_{h \to 0} \frac{f_x(0,h) - f_x(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{\lim\limits_{k \to 0} \frac{f(k,h) - f(0,h)}{k} - \lim\limits_{k \to 0} \frac{f(k,0) - f(0,0)}{k}}{h}$$

$$= \lim_{h \to 0} \frac{\lim\limits_{k \to 0} \frac{\cancel{k} \cdot h \cdot (k^2 - h^2) - 0}{\cancel{k}\,(k^2 + h^2)} - 0}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h} = -1$$

$$(f_y)_x(0,0) = \lim_{h \to 0} \frac{f_y(h,0) - f_y(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{\lim\limits_{k \to 0} \frac{f(h,k) - f(h,0)}{k} - \lim\limits_{k \to 0} \frac{f(0,k) - f(0,0)}{k}}{h}$$

$$= \lim_{h \to 0} \frac{\lim\limits_{k \to 0} \frac{h\cancel{k}\,(h^2 - k^2) - 0}{\cancel{k}(h^2 + k^2)} - 0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = 1$$

Wobei $f(x,y) = \dfrac{x \cdot y \cdot (x^2 - y^2)}{(x^2 + y^2)}$

und $f(0,0) = 0$