(1)

Hints Exercise Sheet (1)

γ be a curve in $\mathbb{R}^n$. Its arclenght is

$$s(t) = \int_{t_0}^{t} \left| \frac{d\gamma}{dt} \right| dt$$

we can always reparametrize by arclenght $\beta(s)$ i.e

$$\beta(s(t)) = \gamma(t) \quad \text{s.t} \quad \left| \frac{d\beta}{ds} \right| = 1$$

- $\tau(s) = \frac{d\beta}{ds} = \frac{d\gamma/dt}{ds/dt}$ "unit tangent vector"
- $k(s) = \frac{d\tau}{ds}$ "curvature vector"

1.a) Show: γ closed curve in $\mathbb{R}^n$, then

$$\int_\gamma |k(s)|\, ds \geq \frac{\pi}{2}$$

pf: let $\gamma : [0,L] \longrightarrow \mathbb{R}^n$, $\dot{\gamma}(s) = \tau(s)$.

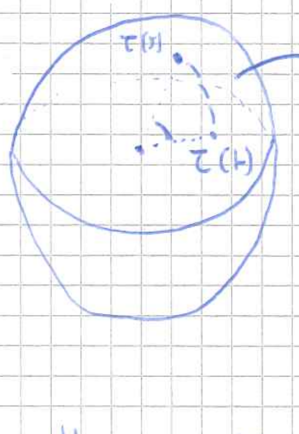
Claim: Consider then function

$$\tau : [0,L] \longrightarrow S^{n-1}, \quad s \longmapsto \tau(s)$$

, then

For any $s \in [0,L]$ $\exists$ a $t \in [0,L]$ s.t

$$\mathrm{dist}_{S^{n-1}} (\tau(s), \tau(t)) \geq \frac{\pi}{2}, \text{ i.e}$$



$\mathrm{dist}_{S^{n-1}} (\tau(s), \tau(t))$ = lenght of the shortest path connecting $\tau(s), \tau(t)$ on S^{n-1}.

= "angle" between $\tau(s), \tau(t)$

pf: Suppose that the assumption is false!
Then $\exists$ $s_0 \in [0,L]$ s.t

$$\mathrm{dist}_{S^{n-1}} (\tau(s_0), \tau(t)) < \frac{\pi}{2} \quad \forall t \in [0,L] \qquad (*)$$