Exercise Sheet 2

Exercise 1

Let G be a locally compact topological group, and let μ be a left Haar measure on G.

(a) G is discrete if and only if $\mu(\{e\}) > 0$.

(b) G is compact if and only if $\mu(G) < \infty$.

Exercise 2

Find an example of a locally compact topological group that contains a Borel set with finite left Haar measure but infinite right Haar measure.

Exercise 3

Prove that the modular function $\Delta\colon G \to (0, \infty)$ (introduced in class) is a continuous homomorphism.

Exercise 4

Consider the locally compact Hausdorff group $G = (\mathbb{R}^n, +)$ where $n \in \mathbb{N}_0$.

(i) Show that $\mathrm{Aut}(G)$, i.e. the group of bijective homomorphisms which are homeomorphisms as well, is given by $\mathrm{GL}(n, \mathbb{R})$.

(ii) Show that $\mathrm{mod}_G : \mathrm{Aut}(G) \to \mathbb{R}_{>0}$ is given by $\alpha \mapsto |\det \alpha|^{-1}$.