Exercisesheet 8

1. **Exercise 2, Winter Exam 2012.** The function $f : (-\rho, \rho) \to \mathbb{R}$ is given by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{n+1}{2^n} x^n.$$

   a) Calculate the radius of convergence ρ of this power series.

   b) Determine a primitive function F of f with $F(0) = 0$. Represent F first as a power series and afterwards as an elementary function.

   c) Use F to find a representation of f as an elementary function.

2. **a)** **Exercise 3, Summer Exam 2008.** Find a power series expansion of the form $\sum\limits_{n=0}^{\infty} a_n x^n$ for the function $f(x) = \frac{2}{1-x+x^2-x^3}$ and determine the radius of convergence.

   Hint: Start by finding a partial fraction decomposition of $f(x)$.

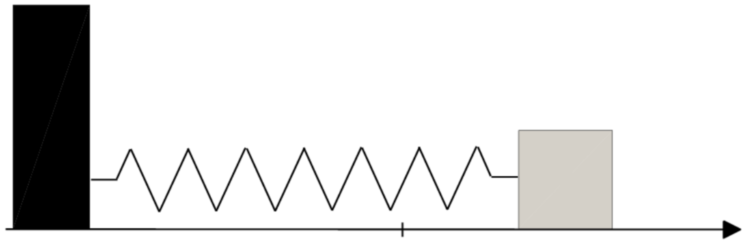
   b) **Exercise 3, Winter Exam 2009.** The coefficients of the power series $\sum\limits_{n=0}^{\infty} a_n x^n$ are given by

$$a_0 = 2 \qquad \text{and} \qquad a_n = \frac{2(n+1)}{n} \cdot a_{n-1}, \quad n \geq 1.$$

   Determine the radius of convergence and show that the power series coincides with the function $f(x) = \frac{2}{(1-2x)^2}$ within the interval of convergence.

Bitte wenden!

3. Harmonic Oscillator:



A mass m, which is connected to a spring with spring constant k and which glides friction-less along the x-axis, satisfies the equation

$$x''(t) = -\frac{k}{m}\, x(t).$$

Here $x(t)$ denotes the deflection from the position of rest $x = 0$ at time t. The motion of the mass is uniquely defined by the specification of so called initial values $x(0) = x_0$, $x'(0) = v_0$ for the deflection and the velocity at the moment $t = 0$.

We now consider a special case of this physical problem and assume in the following that $\frac{k}{m} = 1$, $x_0 = 0$ and $v_0 = 1$. We will solve this special case with the help of power series.

We therefore look for a power series $x(t) = \sum_{k=0}^{\infty} a_k\, t^k$ satisfying

$$x''(t) = -x(t) \quad \text{for all } t > 0 \text{ and} \quad x(0) = 0, \;\; x'(0) = 1.$$

Determine the coefficients a_k.