

## Solutions – Week 11

## SURFACE AREAS, SURFACE INTEGRALS AND COMPUTING THE FLUX THROUGH A SURFACE

1. Compute the surface area of the surfaces described in exercises 1 to 3 in problem set 9.

**Solutions :**  $\sqrt{2}\pi r^2$ ,  $8\pi\left(1 - \frac{\sqrt{2}}{2}\right)$ ,  $4\pi$ .

2. Integrate the given function over the given surface.

- (a)  $f(x, y, z) = z - x$  over the cone from Ex. 1, PS 9.

**Solution :**  $\frac{2\sqrt{2}}{3}\pi r^3$ .

- (b)  $f(x, y, z) = yz$  over the spherical cap from Ex. 2, PS 9.

**Solution :** 0.

- (c)  $f(x, y, z) = x + y + z$  over the surface of the cube cut from the first octant by the planes  $x = a$ ,  $y = a$ ,  $z = a$ .

**Solution :**  $9a^3$ .

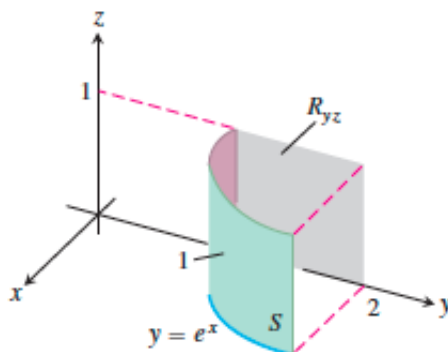
3. Find the outward flux of the field

$$\vec{F}(x, y) = \begin{pmatrix} 2xy \\ 2yz \\ 2xz \end{pmatrix}$$

across the surface described in Ex. 2(c).

**Solution :**  $3a^4$ .

4. Consider the surface that is the green portion of cylinder depicted below. Let  $\vec{n}$  be the normal unit vector **to the surface** pointing away from the  $yz$ -plane.



Determine the flux of the field

$$\vec{F}(x, yz) = \begin{pmatrix} -2 \\ 2y \\ z \end{pmatrix}$$

across  $S$  in direction of  $\vec{n}$ .

**Solution :**  $4(e - 2)$ .