

Problem set – Week 1

LIMITS AND DIFFERENTIAL CALCULUS

1. Consider the function $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} : x \mapsto \frac{x}{x^3+x}$.
 - (a) Draw a picture of graph (f).
 - (b) What is $\text{im}(f)$?
 - (c) What is $\lim_{x \rightarrow 0} f(x)$?
 - (d) Where is f continuous?
2. Give an example of a real function that is differentiable everywhere but its derivative is not continuous at zero. (Warning: this is a challenge question, you will need to think of quite an exotic function!)
3. Compute the first derivative of

$$(a) x^3 e^{-x^3} - x - 3, \quad (b) \frac{\log(\sin^2(x))}{\cos(x)}, \quad (c) \arctan(\sqrt{x}).$$

Compute the second derivative of

$$(d) \log(\log(x)).$$

4. Suppose that a function f is continuous and differentiable in the interval $[0, 1]$. Suppose further that $f(0) = -1$ and $f'(x) \leq 2$ for all $x \in [0, 1]$. What is the largest possible value for $f(1)$?
5. Let x, y be non-negative real numbers such that $x + y = 12$. What is the minimum possible value of $x^2 y$? For which values of x and y is this minimum attained?