Problem set – Week 3

Curves

1. Give a parameterization $r : \mathbb{R} \to \mathbb{R}^2$ of the circle defined by $x^2 + y^2 = 1/4$, in such a way that $|\dot{r}(t)| = 1$ for all t.

2. Consider a circle of radius 1 centered at the point $(0, 1)$, and imagine that this circle is a wheel resting on the x-axis. Imagine that there is a red dot painted on the bottom of the wheel (at the point $(0, 0)$). Give a parameterization of the curve that is traced out by the red dot as the wheel rolls along the x-axis. If the wheel rolls at a constant speed, does the red dot also move at a constant speed?

3. In this problem we find the circumference of a quarter-circle three different ways.

   (a) Find the length of the curve $r : [0, 1] \to \mathbb{R}^2 : x \mapsto \left(x, \sqrt{1 - x^2}\right)$.

   (b) Find the length of the curve $r : [0, \pi/2] \to \mathbb{R}^2 : t \mapsto (\cos t, \sin t)$.

   (c) Check that your answers agree with the formula for the circumference of a circle.

4. Find the length of the curve $r : [0, 10] \to \mathbb{R}^2 : t \mapsto (t \cos t, t \sin t, t)$. Draw the image of this curve.

5. Consider the force $F : \mathbb{R}^2 \to \mathbb{R}^2 : (x, y) \mapsto (x, 1)$ and the curve $r : [0, 1] \to \mathbb{R}^2 : t \mapsto (t, t^t)$. Compute the work of the force F along the curve r.