

Problem set – Week 8

LINEAR ALGEBRA, EXTREMA

1. The following question is from last year's exam:

Consider the equation

$$\begin{pmatrix} 1 & 2 \\ 2 & t \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ t-2 \end{pmatrix}.$$

- (a) For which values of the parameter $t \in \mathbb{R}$ does the above equation have a unique solution $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$?
- (b) Determine this unique solution.
2. The following question is from last year's exam:

A linear mapping

$$\begin{aligned} f: \mathbb{R}^3 &\longrightarrow \mathbb{R}^3 \\ x &\longmapsto Ax \end{aligned}$$

is defined in the standard basis by

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

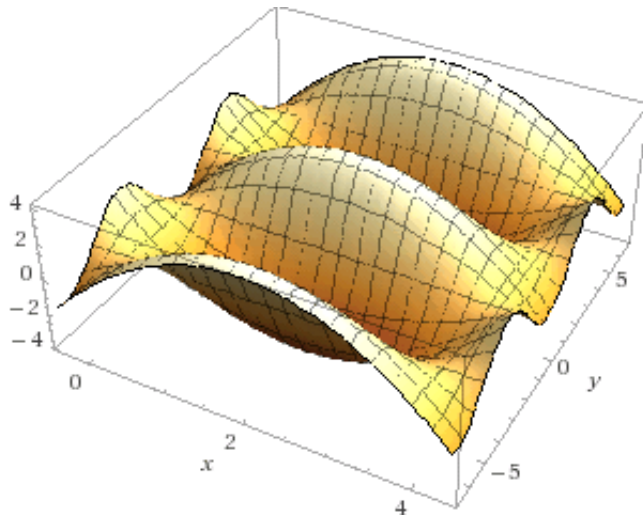
- (a) Compute the eigenvalues of the matrix A . For each eigenvalue of A determine one corresponding eigenvector.
- (b) Determine a diagonal matrix D and a corresponding transformation matrix T with

$$T^{-1}AT = D.$$

3. What are the determinants of the following matrices?

$$(a) \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{pmatrix} \qquad (b) \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 1 & 1 & 2 & 2 \\ 3 & 3 & 4 & 4 \end{pmatrix}$$

4. Find the absolute extrema of the surface $f(x, y) = (4x - x^2) \cos(y)$ on the rectangular plate $1 \leq x \leq 3$, $-\pi/4 \leq y \leq \pi/4$.



5. A flat circular plate P of radius 1 is heated (including the boundary of the plate) so that the temperature at the point $(x, y) \in P$ is

$$T(x, y) = x^2 + 2y^2 - x.$$

Find the temperatures at the hottest and coldest points on the plate.

6. Find three positive real numbers whose sum is 9 and whose sum of squares is a minimum.