Homotopical and Higher Algebra: detailed plan

Week 1

Symmetric monoidal categories

Definition and examples of monoidal categories (Sets, Vect, k-mod, Alg, ...). Draw coherence diagrams and state Mac Lane coherence theorem. Same for symmetric monoidal categories (ex: chain complexes). Symmetric monoidal functors (ex: homology).
References: [9, Ch. XI] and [7, §3.2].

The symmetric monoidal category $n\mathsf{Cob}$, and $n\mathsf{TFTs}$

General constuction (oriented and unoriented versions). Detailed description of $1\mathsf{Cob}$ and $1\mathsf{TFTs}$: classification theorem for 1-dim. manifolds with boundary (both versions).
References: [7, §1.1, 1.2 & 1.3] and [8, §1.1].

Week 2

Duality in monoidal categories

Left and right duals in a monoidal category. Equivalence of the two notions in the symmetric monoidal case. Examples of dualizable objects in Sets, Vect, k-mod and Alg. Property preserved by monoidal functors. Any object is dualizable in $n\mathsf{Cob}$.
Reference: [8, p.38]

Presentation of $1\mathsf{Cob}$ by generators and relations

Generators and relations for (symmetric) monoidal categories. Give a presentation by generators and relations for $1\mathsf{Cob}$. Conclude that $1\mathsf{TFTs}$ with values in $\mathcal{C}$ are dualizable objects in $\mathcal{C}$ (in other words, $1\mathsf{Cob}$ *is the free symmetric monoidal category generated by a single dualizable object*).
Reference: [7, §1.4]

Week 3

Explicit description of $2\mathsf{Cob}$

Classification theorem for oriented 2-dim manifolds with boundary. Pant decomposition and generators of $2\mathsf{Cob}$. Relations for $2\mathsf{Cob}$.
References: [7, §1.4] and [1].

Frobenius algebras and 2TFTs

Definition and examples of Frobenius algebras. Frobenius algebras are oriented 2TFTs.
References: [7, Ch. 2 and §3.3] and [1].

Week 4

Extending down TFTs

Classifying 3-manifolds. Problem in finding generators (i.e. problem of computing 3TFTs). Solution: manifolds with boundaries as generators. Extending down TFTs. Heuristic definition of 2-extended and fully extended TFTs. Need for higher categories.
References: [8, §1.2]

Bicategories

Definition, examples and first properties of bicategories (Cat, sets with correspondences, algebras with bimodules, $2\mathsf{Cob}^{\mathsf{ext}}$, the classifying 2-category of a monoidal category).
References: [3] and [10, Appendix B].

Week 5

Adjoints in bicategories

Adjoints in a bicategory (ex: usual adjoints in Cat, and dualizable objects in a monoidal category). Every 1-morphism in $2\mathsf{Cob}^{\mathsf{ext}}$ has left and right adjoints.
Reference: [8, pp.39-42]

Symmetric monoidal bicategories

Disjoint union in $2\mathsf{Cob}^{\mathsf{ext}}$ is an additional structure. Extract from it the definition of a symmetric monoidal bicategory. Main examples of symmetric monoidal bicategories (Cat, sets with correspondences, algebras with bimodules, $2\mathsf{Cob}^{\mathsf{ext}}$, ...).
References: [10, Ch. 2].

Week 6

2-dualizable objects

Define 2-dualizable objects as objects having duals for which (co)evaluation have adjoints. Example: 2-dualizable objects in Alg^2 and relation with separable Frobenius algebras.
References: [8, pp.39-42] and [10, §A.3].

2-extended 2TFTs

Description of $2\mathsf{Cob}^{\mathsf{ext}}$ by generators and relations. Relation with the free symmetric monoidal bicategory generated by a single 2-dualizable object.
References: [10, Ch. 3] amd [8, pp.92-94].

Week 7

Extending up: ∞-categories

Summary of what we've done so far.
Explanation for why we need higher invertible arrows.
Overview of models for higher categories.

Week 8

Model categories

Definition and examples. Yoga of derived functors. Consider the main example of chain complexes.
Reference: [6].

Models for ∞-groupoids

Model structure on topological spaces and on simplicial sets. Kan complexes. Geometric realization and Quillen equivalence.
Reference: [6].

Week 9

Models for $(\infty, 1)$-categories

Topological and simplicial categories. Quasi-categories and weak Kan complexes. Complete Segal spaces and Segal categories.
Reference: [4]

Quillen equivalences

Sketch of proof that all models are equivalent.
References: [4, §7] and [5].

Week 10

The $(\infty, 1)$-category $\mathtt{1Cob}_\infty$

Description of $\mathtt{1Cob}_\infty$ as a complete Segal space.
Reference: [8, §2.2].

The homotopy category of $\mathtt{1Cob}_\infty$ is $\mathtt{1Cob}$

Topological and differentiable structures. Contractibility of the space of choices.

Week 11

Symmetric monoidal structure on $\mathtt{1Cob}_\infty$

Discuss symmetric monoidal structures on $(\infty, 1)$-categories. Examples: Chain complexes and $\mathtt{dg\text{-}Alg}$.

$1\mathsf{TFT}_\infty$'s

Symmetric monoidal structure on $1\mathsf{Cob}_\infty$. Description of $1\mathsf{TFT}_\infty$'s with values in chain complexes and dg-Alg.
Reference: [8, beginning of §4.2]

Week 12

Detailed study of $2\mathsf{TFT}_\infty$'s. In particular, the homotopy category of $2\mathsf{Cob}_\infty$ is $2\mathsf{Cob}^{ext}$.

Week 13

Lurie's main theorem (fully dualizable objects).

References

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