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Student Seminar: Monstrous Moonshine

ETH Zürich, Herbstsemester 2015

Organizer: Dr. Christoph Keller, HG G 23.2, christoph.keller@math.ethz.ch

Assistant: Claude Eicher, HG F 27.6, claude.eicher@math.ethz.ch

Time and Location: Wed 16-18, HG F 26.3

Goal: The goal of this seminar is to understand the equation

$$196884 = 196883 + 1 \tag{1}$$

The left-hand side of (1) is the first non-trivial coefficient of the modular invariant j function. On the right hand-side are the dimensions of the smallest irreducible representations of the largest sporadic simple group, the Monster group $\mathbb{M}$.

Content: The talks will give an overview over the topics necessary to understand (1), namely modular forms, lattices, the classification of finite simple groups, and the theory of Vertex Operator Algebras. The goal is not to give a complete and detailed understanding of those topics, but to provide enough information to understand what (1) means, why it is surprising, and to give a general idea of how this mysterious relation can be explained.

Prerequisites: Algebra I and II. Some familiarity with modular forms and Lie algebras is helpful, but not crucial: all necessary concepts will be introduced in the early talks.

Literature: This seminar will be heavily based on Terry Gannon's book [1]. For a briefer overview see also [2] and part 2 of [3].

Organization: Try to get in touch with your mentor as early as possible to discuss your talk. One week before your talk, meet your mentor after the regular seminar session to present an outline of your talk. Attend at least 11 of the 13 sessions.

Schedule of Talks

Talks marked with * are somewhat more advanced.

1 Modular Forms I

Speaker: Nina Aerni
Topics: Basic Definitions
References: Section 1 of [4].
Date: Week 2
Mentor: Christoph Keller

2 Modular Forms II

Speaker: Nicolas Kirchmayer
Topics: Eisenstein Series and the Klein j function.
References: Section 2 of [4].
Date: Week 3
Mentor: Christoph Keller

3 Lattices I

Speaker: Lukas Böke
Topics: Basic defintions: Integral lattices, even lattices, dual lattices, unimodular lattices.
References: Chapters 1-2 in [5]
Date: Week 4
Mentor: Christoph Keller

4 Lattices II

Speaker: Dénes Heins
Topics: Theta functions. Poisson Resummation. Lattice theta functions. Hecke-Schoenberg.
References: Section 3 of [4]
Date: Week 4
Mentor: Christoph Keller

5 Lattices III

Speaker: Christian Schneebeli
Topics: Gluing constructions. Niemeier Lattices in $d = 24$. Golay Code. Leech lattice.
References: Chapters 4,16,2,3 in [5],[6]
Date: Week 5
Mentor: Christoph Keller

6 Group Theory I

Speaker: Laurin Köhler-Schindler
Topics: Normal subgroups. Simple Groups. Jordan-Hölder Theorem.
References: I.3 of [7]
Date: Week 6
Mentor: Claude Eicher

7 Group Theory II

Speaker: Manuel Akeret
Topics: Representations of finite groups. Character formulae.
References: XVIII.1 – XVIII.5 of [7]
Date: Week 6
Mentor: Claude Eicher

8 Lie Theory I

Speaker: Elyasa Erdogan
Topics: Lie algebras. Jacobi identity. Classification. Highest weight representations.
References: Sections 1.4 and 1.5 in [1]. Sections 13.1 and 13.2 in [8]. [9].
Date: Week 7
Mentor: Claude Eicher

9 Lie Theory II*

Speaker: Viktor Burghardt
Topics: Affine Lie Algebras. The Virasoro Algebra.
References: Section 3 in [1]. Section 14 in [8].
Date: Week 8
Mentor: Claude Eicher

10 Classification of simple groups I

Speaker: Michael Näf
Topics: Give an overview over the infinite families.
References: Sections 2–4 in [10]
Date: Week 9
Mentor: Claude Eicher

11 Classification of simple groups II

Speaker: Elena Asoni
Topics: Give an overview over the sporadic groups.
References: Section 5 in [10]
Date: Week 9
Mentor: Claude Eicher

12 The Monster Group*

Speaker: Selim Bieri
Topics: Construction of the Monster. Representations.
References: Chapter 29 of [5]. Section 5.8 in [10]
Date: Week 10
Mentor: Christoph Keller

13 Vertex Operator Algebras I*

Speaker: Simon Päpcke
Topics: Basic definitions: Formal power series. The δ series. Binomial convention. Axioms of VOA. Weak Commutativity. Correlation functions.
References: 5.1 and 5.3.1 in [1]. Section 2 in [11]. Chapters 1–3 in [12]
Date: Week 11
Mentor: Christoph Keller

14 Vertex Operator Algebras II*

Speaker: Lluís Usó i Cubertorer
Topics: Examples: Free boson, Heisenberg algebra. Lattice algebras.
References: 5.2.2 in [1]. Section 3 in [11]. Sections 6.3–6.5 in [12]
Date: Week 12
Mentor: Christoph Keller

15 Vertex Operator Algebras III*

Speaker: Peter Zenz
Topics: Modules of VOAs. Rational CFTs. Self-dual CFTs. Zhu's Algebra. Modular invariance.
References: 5.3 in [1]. [13].
Date: Week 13
Mentor: Claude Eicher

16 The Monster VOA*

Speaker: Manuel Göggel
Topics: $\mathbb{Z}_2$ orbifolds. FLM construction.
References: 4.3.4, 5.3.6, 7.2 in [1].
Date: Week 14
Mentor: Christoph Keller

References

[1] T. Gannon, *Moonshine beyond the Monster*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Cambridge, 2006.

[2] T. Gannon, "Monstrous moonshine: The First twenty five years," arXiv:math/0402345 [math-qa].

[3] T. Gannon, "Monstrous moonshine and the classification of CFT," arXiv:math/9906167 [math].

[4] D. Zagier, "Elliptic modular forms and their applications," in *The 1-2-3 of modular forms*, Universitext, pp. 1–103. Springer, Berlin, 2008. http://dx.doi.org/10.1007/978-3-540-74119-0_1.

[5] J. H. Conway and N. J. A. Sloane, *Sphere packings, lattices and groups*, vol. 290 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, New York, third ed., 1999. http://dx.doi.org/10.1007/978-1-4757-6568-7. With additional contributions by E. Bannai, R. E. Borcherds, J. Leech, S. P. Norton, A. M. Odlyzko, R. A. Parker, L. Queen and B. B. Venkov.

[6] R. A. Wilson, "The leech lattice,". http://www.maths.qmul.ac.uk/~raw/talks_files/Leech.pdf.

[7] S. Lang, *Algebra*, vol. 211 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, third ed., 2002. http://dx.doi.org/10.1007/978-1-4613-0041-0.

[8] P. Di Francesco, P. Mathieu, and D. Sénéchal, *Conformal field theory*. Graduate Texts in Contemporary Physics. Springer-Verlag, New York, 1997. http://dx.doi.org/10.1007/978-1-4612-2256-9.

[9] W. Fulton and J. Harris, *Representation theory*, vol. 129 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1991.

[10] R. A. Wilson, *The finite simple groups*, vol. 251 of *Graduate Texts in Mathematics*. Springer-Verlag London, Ltd., London, 2009. http://dx.doi.org/10.1007/978-1-84800-988-2.

[11] C. Dong, "Introduction to vertex operator algebras. I," *Sūrikaisekikenkyūsho Kōkyūroku* no. 904, (1995) 1–25. Moonshine and vertex operator algebra (Japanese) (Kyoto, 1994).

[12] J. Lepowsky and H. Li, *Introduction to vertex operator algebras and their representations*, vol. 227 of *Progress in Mathematics*. Birkhäuser Boston, Inc., Boston, MA, 2004. http://dx.doi.org/10.1007/978-0-8176-8186-9.

[13] Y. Zhu, "Modular invariance of characters of vertex operator algebras," *J. Amer. Math. Soc.* **9** no. 1, (1996) 237–302.