Generalized Kuhn-Tucker Conditions for Stochastic Irreversible Investments with Limited Resources

Maria B. Chiarolla
Department of Methods and Models for Economics, Territory and Finance,
University of Rome 'La Sapienza' , Italy
maria.chiarolla@uniroma1.it

Giorgio Ferrari
Department of Methods and Models for Economics, Territory and Finance,
University of Rome 'La Sapienza', Italy
giorgio.ferrari@uniroma1.it

Frank Riedel
Institute of Mathematical Economics (IMW), University of Bielefeld,
Germany
friedel@wiwi.uni-bielefeld.de

In this paper we study a continuous-time, optimal stochastic investment problem with limited resources in a market with two firms. The investment processes are subject to a dynamic stochastic constraint. Rather than using a dynamic programming approach, we exploit the concavity of the profit functional to derive some necessary and sufficient first order conditions for the corresponding *Social Planner* optimal policy. Our conditions are a stochastic infinite-dimensional generalization of the Kuhn-Tucker Theorem. As a sub product we obtain an enlightening interpretation of the first order conditions in Bank [2] for a single firm profit maximization problem.

In the infinite-horizon case with Cobb-Douglas production functions our method allows the explicit calculation of the optimal policy in terms of the base capacity process, i.e. the unique solution of Bank and El Karoui's Representation Theorem [1].

References

[1] P. Bank and N. El Karoui *A Stochastic Representation Theorem with Applications to Optimizazion and Obstacle Problems.* The Annals of Probability **32** (2004): 1030-1067.

[2] P. Bank. *Optimal Control under a Dynamic Fuel Constraint.* SIAM Journal of Control and Optimization **44** (2005): 1529-1541.

[3] M.B. Chiarolla and U.G. Haussmann. *On a Stochastic Irreversible Investment problem.* SIAM Journal of Control and Optimization **48** (2009): 438-462.

[4] I. Karatzas. *Probabilistic Aspects of Finite-Fuel Stochastic Control.* Proc. Nat'l Acad. Sci. USA **82** (1985): 5579-5581.

[5] F. Riedel and X. Su, *On Irreversible Investment.* Finance and Stochastics **14** (2010).