FIM Nachdiplomvorlesung

Alexander Shnirelman (Concordia University)

Some problems of Mathematical Fluid Dynamics

27 September to 20 December 2016

Tuesdays, 10:15 - 12:00

HG G 43, ETH Zürich, Rämistrasse 101

Abstract

Ideal incompressible fluid is one of the most fundamental models in the continuum mechanics. The fluid flow is described by the Euler-Lagrange equations which, in spite of their apparent simplicity, are among the toughest in the whole mathematics. No wonder that the study of these equations involves many different mathematical structures, and requires a stereoscopic approach to capture the full picture. In these lectures I'll propose various viewpoints on the fluid. Here are the topics of the lectures.

1. Analyticity properties of the flows of the ideal incompressible fluid. Complex singularities, their persistence and propagation.

2. Singularities of solutions, their propagation and evolution. Exponential map as an elliptic paradifferential operator. Microlocal integrals and Liapunov functions. Global geometrical structure of the group of volume preserving diffeomorphisms and the exponential map.

3. Two-point problem for the Euler-Lagrange equations. Surjectivity of the exponential map in 2-d, and pathologies in 3-d.

4. Braids and flows. Length minimizing braids and weak solutions in 2-d. Minimal folded braids and the inverse cascade solutions.

5. Generalized flows and weak solutions of the Euler equations. Models of turbulent cascade.

6. Long-time behavior of solutions of the Euler equations and the existence of a non-trivial attractor.

www.fim.math.ethz.ch/lectures

FIM
Forschungsinstitut für Mathematik

ETH zürich