FIM Nachdiplomvorlesung

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Topics in modern analytic number theory

The many faces of the modular group

25 September to 18 December 2019

Wednesdays, 13:15 - 15:00

HG G 43, ETH Zürich, Rämistrasse 101

Abstract

This series of lectures will treat a number of special topics all of which involve some aspect of the modular group SL(2,Z). Many have as their natural starting point the class number formulas of Dirichlet for binary quadratic form or, equivalently, ideal classes of quadratic number fields. Most involve L-functions of various kinds and modular forms and their generalizations.

These topics include:

1. New class number formulas, both for quadratic forms and higher degree forms, for example binary cubic forms
2. Properties of geometric objects associated to the modular group, including cm points, closed geodesics and infinite volume surfaces having closed geodesics as boundaries.
3. The interaction between flat metrics and the hyperbolic metric, for instance the curvature of closed geodesics with respect to a flat metric.
4. Quasimorphisms of the modular group and modular cocycles. Linking numbers of modular knots.
5. Markov forms and modular billiards
6. Connections with Diophantine approximation and the geometry of numbers.

I will attempt to provide enough background and historical context for each topic to make them accessible to non-experts, but hope to give enough “new” material to stimulate the interest of the experts as well. I will also give suggestions about areas for further research.

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