

FIM Nachdiplomvorlesungen

HS 2022

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A PDE approach to mean-field disordered systems

05 October - 21 December 2022 | Wednesdays, 10:15 - 12:00 | HG 43

The goal of statistical mechanics is to describe the large-scale behavior of collections of simple elements, often called spins, that interact through locally simple rules and are influenced by some amount of noise. We will discuss three classes of such models, of increasing difficulty, and will rely on a common PDE approach to study each of them.

The first model we will study is the very simple Curie-Weiss model, in which every spin interacts with every other spin and has a preference for being aligned with the others. This model can be solved in a variety of ways, but will be used to develop our toolkit based on the study of certain Hamilton-Jacobi equations that naturally arise.

We will then turn to a more challenging class of models coming from statistical inference. We will focus on a setup in which we observe a noisy version of a large rank-one matrix. We will compute the information-theoretic limit to the recovery of this matrix based on the PDE techniques introduced earlier.

We will finally discuss spin-glass models, in which the local interactions between the spins are disordered. One of the core motivations for the development of the techniques presented here is to uncover the behavior of models in which spins can be of different types, such as for instance when the spins are organized over two layers, and only have direct interactions across layers. While the understanding of this class of models is still very limited, I will present some progress towards this goal.

The prerequisites for these lectures are basic measure theory and probability theory. No prior knowledge of PDE theory will be assumed.

<https://math.ethz.ch/fim/activities/nachdiplom-lectures.html>

